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Metric dimensions, generalized integrations, cauchy transform, and riemann boundary-value problem on nonrectifiable arcs

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Abstract

We consider a nonrectifiable Jordan arc Γ on the complex plane ℂ with endpoints a1 and a2. The Riemann boundary-value problem on this arc is the problem of finding a function Φ(z) holomorphic in ℂ \ Γ satisfying the equality, where Φ±(t) are the limit values of Φ(z) at a point t ∈ Γ \ {a1, a2} from the left and from the right, respectively. We introduce certain distributions with supports on nonrectifiable arc Γ that generalize the operation of weighted integration along this arc. Then we consider boundary behavior of the Cauchy transforms of these distributions, i.e., their convolutions with (2πiz)-1. As a result, we obtain a description of solutions of the Riemann boundary-value problem in terms of a new version of the metric dimension of the arc Γ, the so-called approximation dimension. It characterizes the rate of best approximation of Γ by polygonal lines. © 2013 Springer Science+Business Media New York.

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